THE POLYKARP KUSCH LECTURE SERIES

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Conflicts in Supply Chains and Contracts that Restore Efficiency

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What is a Supply Chain?

AIMS Video (0:20 through 2:35)

https://youtu.be/xl3NJ0HC86k

What is a Supply Chain?

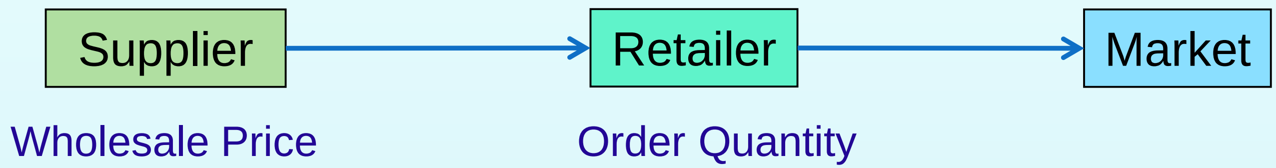
Our Focus Today: The Big Picture

- We consider a simple supply chain of a Supplier and a Retailer who each make independent pricing and ordering decisions.

- We consider only cash and product flows.
- Everyone has full information about costs and demands.
- The Supplier and the Retailer make independent decisions in a decentralized fashion, which are often not the best decisions for the total supply chain!
- We can design contracts that align each one's objective with the objective of the total supply chain.

Conflicts in the Supply Chain Cause Inefficiency

- The Supplier sells through a Retailer

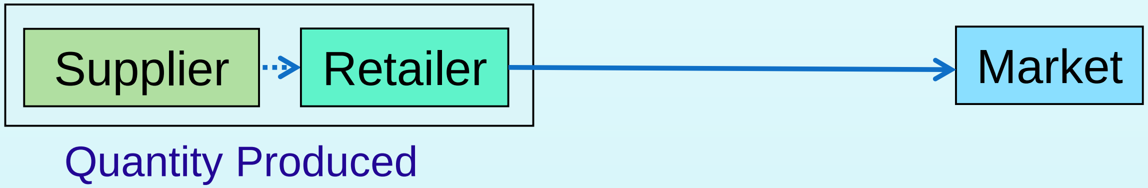


- The Supplier sets the wholesale price to maximize his profit.
- The Retailer, in response, decides on an order quantity to maximize his profit.
- Their self-serving actions do not maximize the profit of the entire supply chain.
- This resulting inefficiency is due to Double Marginalization. Why?
  1. The Supplier puts up a margin over his cost to set the wholesale price.
  2. The Retailer earns a margin that equals the difference between the retail price and the wholesale price he pays.

"Margin" appears twice in the explanation! Double Marginalization, Haha.

Centralized Solution as a Benchmark

- In order to assess the loss of efficiency in the (decentralized) Supply Chain, we ask the following question: What would be the maximum profit if the Supplier and the Retailer were one company?



- It turns out that this gives a higher profit than the decentralized solution, and thus serves as a benchmark.
- Typically, the centralized solution produces a higher quantity than that ordered by the Retailer in the decentralized solution.
- This is because the Retailer does not want to take the risk of getting stuck with too many unsold products given the uncertain demand he faces.

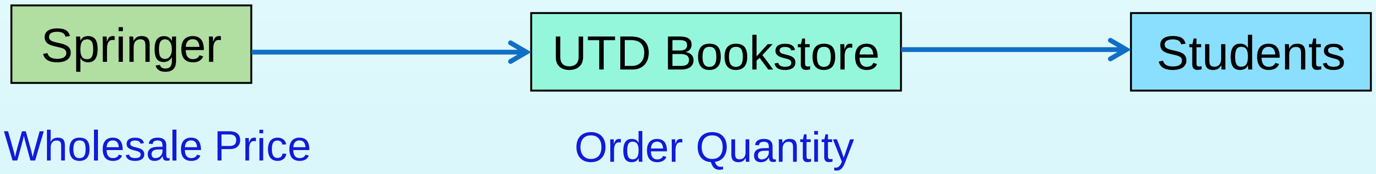
A Buyback Contract Restores Efficiency

- In a Buyback Contract, the Supplier offers to buy the unsold products at an agreed upon price.
- This reduces the Retailer's risk and encourages him to order a higher quantity.
- The higher the buyback price, the higher the quantity ordered.
- There is a buyback price along with the wholesale price at which the Retailer will order exactly the quantity in the centralized solution, thus restoring efficiency.

We will now illustrate all of these concepts by way of a simple example.

A Simple Illustrative Example

- Consider the case of the UTD bookstore ordering a number of textbooks from Springer, the publisher, for an upcoming course.



- We have the following data for ease of illustration:

Production cost of a book (includes royalty, etc.)	$99
Retail Price	$135
Salvage value if not sold	$78
Enrollment in the course	10-15

A Simple Illustrative Example (cont.)

- The randomness in demand arises from factors such as uncertain class enrollment and students buying books from other suppliers.

  Past experience suggests:

Demand	Probability
10	15%
11	22%
12	30%
13	20%
14	9%
15	4%

- We solve this problem as a Leader-Follower game, called a Stackelberg Game, named after Heinrich Freiherr von Stackelberg, *Market Structure and Equilibrium*, 1934.
- What is a Stackelberg Equilibrium?

Nash Equilibrium

- Before we get into the Stackelberg equilibrium, let us discuss the more popular Nash Equilibrium by John Nash (1950), whom you might know from the movie *A Beautiful Mind*.

- In a game of two or more players, a set of decisions by the players is a Nash Equilibrium if no player can do better by changing his decision, while the others stay with their decisions.
- If a player can do better by changing his decision, knowing the decisions of the others and treating them as set in stone, then the set of decisions is not a Nash Equilibrium.

Let us now review the bar scene in *A Beautiful Mind*.

The Bar Scene in *A Beautiful Mind*

A Beautiful Mind video (0:00 through 2:45)

https://youtu.be/CemLiSI5ox8

Nash Equilibrium (cont.)

- In the movie, Nash thanks the blonde woman for his epiphany, and goes home to write his 26 page thesis, titled *Non-Cooperative Games*, where he develops the concept of the Nash Equilibrium.

- The four friends, I suppose, act on Nash's advice and they all choose the brunette women.

- Question: Do their decisions form a Nash Equilibrium?

  Who is for YES? Who is for NO?

- Answer: No, this is not a Nash Equilibrium, since each friend could gain by going for the blonde woman while the others stay with the brunette women.

Nash Equilibrium (cont.)

- Consider now a simpler version of the scene with two men, **Tom** and **Dick** and two women, Emily and Linda. Both men prefer Emily to Linda and no one gets any if they both choose the same girl.

Dick Tom	Emily	Linda
Emily	(**Emily**, **Emily**)	(**Emily**, **Linda**)
Linda	(**Linda**, **Emily**)	(**Linda**, **Linda**)

- Two Nash Equilibria:
  1. Tom-Emily, Dick-Linda
  2. Tom-Linda, Dick-Emily
- In (1.), if Tom deviates by going for Linda, he suffers by getting none. Likewise, if Dick deviates by going for Emily, he suffers the same fate.
- A similar argument holds for (2.).

Stackelberg Equilibrium

- Let us have Tom as the leader and Dick as the follower.
- This sequential game is solved by using backward induction. First, we obtain the best response function by Dick for each action by Tom. Then we get Tom's best action.

Response Function - Purple

Tom's Decision	Dick's Response
Emily	Linda
Linda	Emily

Response Function - Yellow

Tom's Decision	Dick's Response
Emily	Emily
Linda	Emily

Response Function - Blue

Tom's Decision	Dick's Response
Emily	Linda
Linda	Linda

- Dick's three possible response functions are shown as Purple, Yellow, and Blue. Clearly, Purple is his best response function.
- Given Dick's best response function (Purple), it is clear that Tom's best action is to choose Emily. And from Dick's Purple response function, he gets Linda.
- So the Stackelberg Equilibrium is: Tom-Emily, Dick-Linda.
- Clearly, if Tom deviates he loses, given the best response function of Dick in Purple. And if Dick changes his response to Yellow or Blue, given that Tom has chosen Emily, he loses.

Differences Between Nash and Stackelberg

- Obvious difference: Nash involves simultaneous decisions and Stackelberg involves sequential decisions.

- It is also clear that Tom has the first-mover advantage in Stackelberg, as he gets Emily.

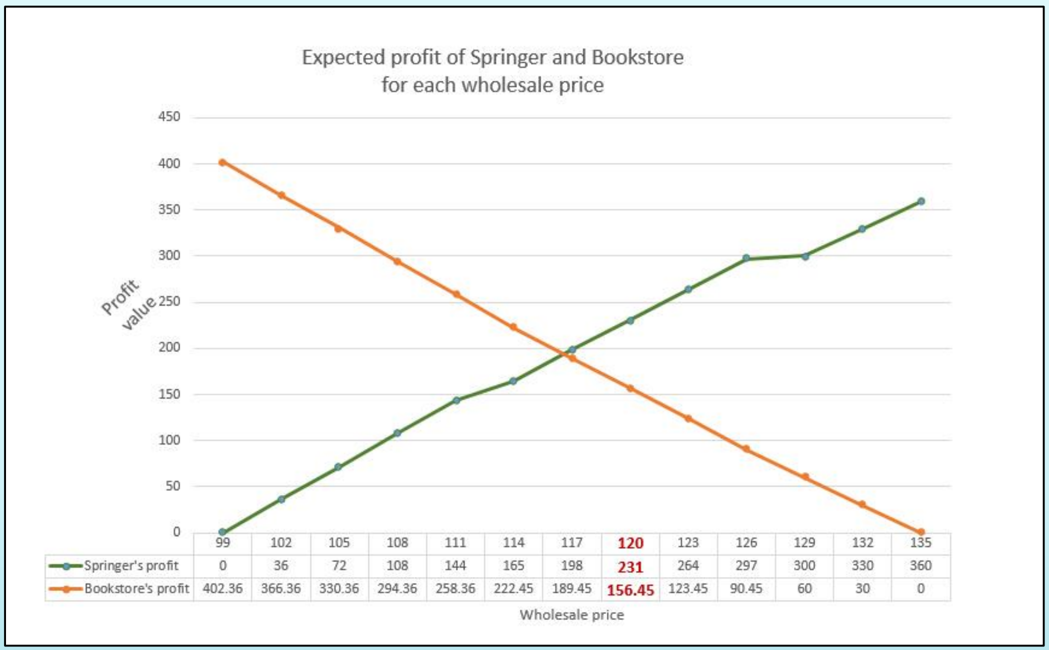
- Subtle difference: Dick's deviation for checking the Nash Equilibrium is to change his action (going from the woman in the equilibrium to another), whereas his deviation for checking the Stackelberg Equilibrium is to change his response function (going from Purple to another color).

Bookstore's Problem: Finding Its Best Response

- For any given wholesale price offered by Springer, the bookstore would order a number of books that maximizes its "expected" profit to meet the uncertain demand for the book.
- The expected profit equals the revenue from books sold and unsold, less the purchase cost of the order paid to Springer.

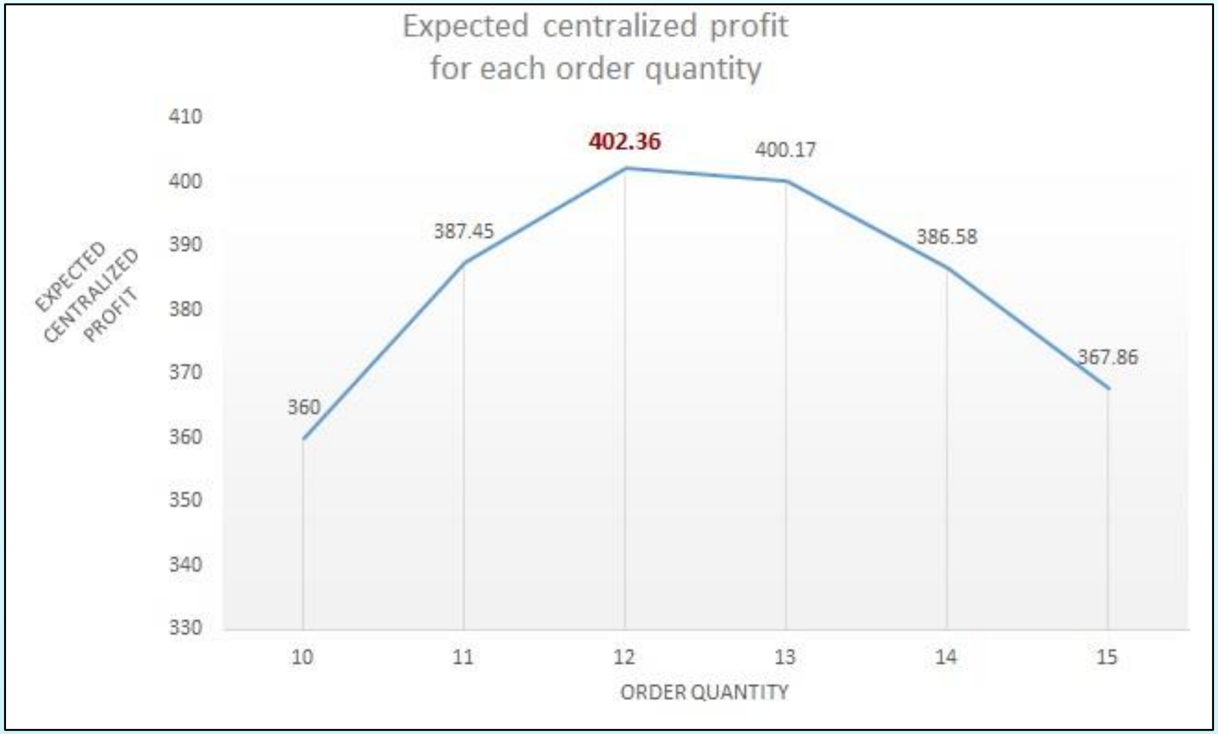
Springer's Problem

- To determine the wholesale price, Springer has the following information:
  - What the bookstore will order for each wholesale price (from the previous slide).
  - The store's reservation profit is the best alternative opportunity. Let us set this to be $156.
  - Springer will offer the wholesale price that maximizes its profit while giving the bookstore no more than its reservation profit. This is also called the leader's first-mover advantage.



- Springer will set the wholesale price at $120. The store will order 11 books. The total supply chain profit is $387.

Benchmark: The Expected Centralized Profit



- Clearly, the centralized firm will produce 12 books with the expected profit of $402. This profit is higher than $387 in the decentralized case. So the profit loss due to Double Marginalization is $15.

A Buyback Contract that Restores Efficiency

- Springer's problem is to offer a wholesale price and a buyback price for the returned unsold books that will give the store no more than its reservation profit.

- With the wholesale price of $120 and buyback price of $100, Springer's profit is $246, the store's profit is $156, and the total supply chain profit equals the benchmark $402.

- With the wholesale price of $118 and buyback price of $93, Springer's profit is $233, the store's profit is $169, and the total supply chain profit equals the benchmark $402.

- In both of these solutions, neither Springer nor the store gets less profit than in the solution without the buyback contract.

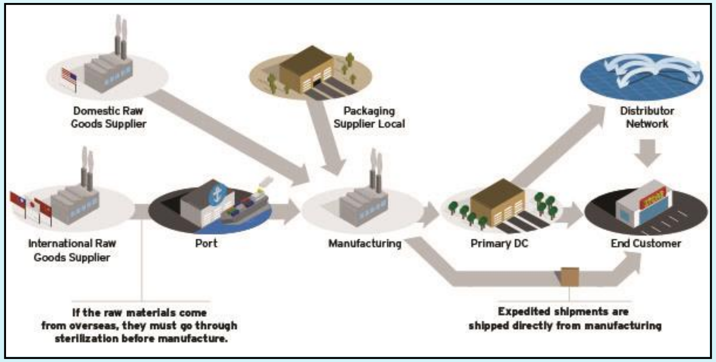
- For example, in the second solution, Springer's profit increases by $2 and the store's profit increases by $13. So this is how they split the additional $15 in profit in the second solution.

Insights Gained

- The best centralized order quantity is higher than the equilibrium order quantity by the bookstore. This is because the bookstore does not want to take too much risk of getting stuck with unsold books in the case when the realized demand is low.
- This issue leads to a loss of efficiency.
- In the buyback contract, Springer offers to buy back the unsold books at a price better than the salvage value of the book (which in some cases may be zero or even negative). This reduces the store's risk, allowing it to order more than that in the absence of the contract.
- We can determine a number of possible wholesale price / buyback price combinations, that result in the store ordering exactly the centralized optimal quantity and thereby achieving the first-best benchmark profit.
- Moreover, each optimal pair also determines the respective profits of Springer and the store. The combination chosen depends on their respective bargaining powers.

Extensions

- Customers can also be decision makers. Retail price can be a decision variable and customers can decide to buy or not to buy.
- There may be a number of competing supply chains.
- More generally, a supply chain in reality is a supply network.



- Asymmetric information: this case arises when the leader ("Principal") does not have full information about the follower ("Agent").

- Multiple Periods (my current work)

Multi-Period Supply Chains

- For simplicity, consider just two periods.
- Now there is a state involved. E.g., what is not sold in the first period can be carried as inventory in the second period. This inventory will be a random variable that becomes known after the demand is realized.
- Depending on the sequence in which decisions are made, we have different types of equilibria.

Types of Stackelberg Equilibria in Two-Period Supply Chains

- *Open-Loop Solution*
  - The Supplier announces the wholesale prices in periods 1 and 2 at the beginning of the game.
  - The Retailer's best response takes both of these prices to decide on the order quantities in each period.
  - The leader continues to have the first-mover advantage.
  - Drawback: The solution does not take into account the realization of the demand in period 1.
  - Drawback: The solution is not time-consistent. This means the Supplier has an incentive in period 2 to renege on the wholesale price announced for period 2 at the beginning of the game.
  - For this to work, the Supplier has to commit to his announced prices. It is therefore also called a Game with Full Commitment.

Types of Stackelberg Equilibria in Two-Period Supply Chains (cont.)

- *Feedback Stackelberg Solution*
  - The Supplier announces only the first-period wholesale price at the beginning of period 1. But then a solution in period 1 must anticipate what would happen in period 2.
  - In period 2, the Supplier announces the second period wholesale price after observing the inventory at the beginning of period 2.
  - The Retailer responds to the Supplier's wholesale price period-wise to obtain his period-wise order quantity.
  - Both take into account the state observed in the second period.
  - The solution is time-consistent.
  - Drawback: The leader only has a period-wise first-mover advantage.

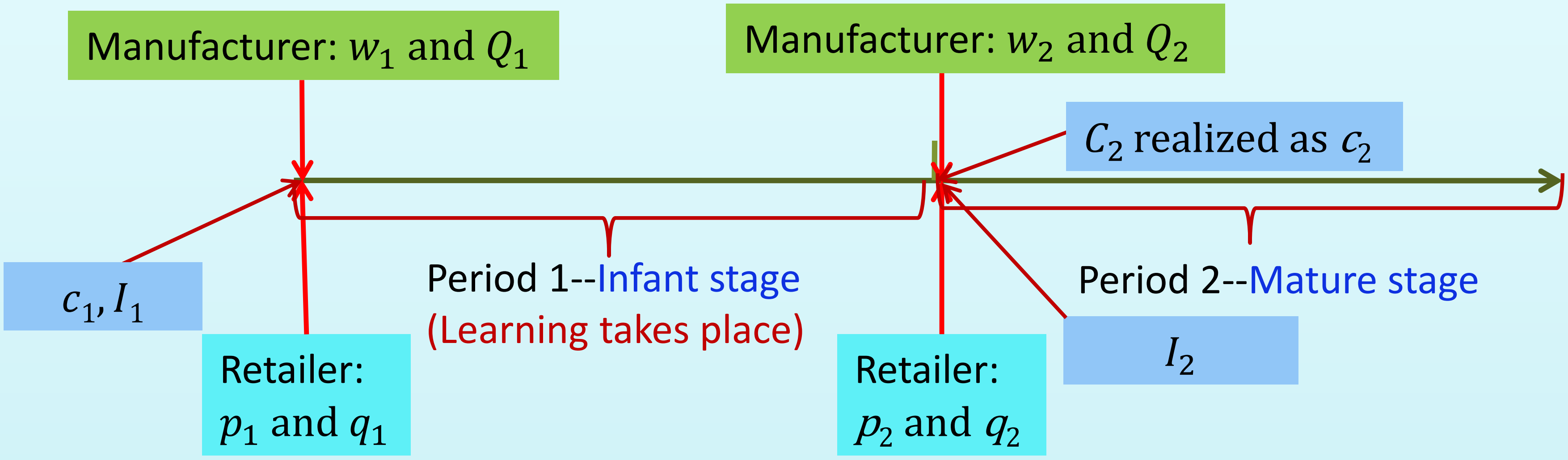
Dynamic Pricing, Production, and Channel Coordination with Stochastic Learning

Li, Sethi and He, *Production and Operations Management*, 24(6), pp. 857-882, 2015.

- Production Cost Learning: The cost of production in the second period declines in proportion to the quantity produced in the first period.

A Two-Period Problem: Sequence of Events

- Manufacturer as the leader, Retailer as the follower



States and Functional Dependence of Decisions

- State: $c_1, I_1; C_2, I_2$
- Manufacturer's Decision: $w_1, Q_1; w_2(C_2, I_2), Q_2(C_2, I_2)$
- Retailer's Decision: $p_1(w_1, Q_1), q_1(w_1, Q_1); p_2(C_2, I_2, w_2, Q_2), q_2(C_2, I_2, w_2, Q_2)$

Conclusions

- Double Marginalization becomes more severe with learning.
- Two major drivers for inventory carryover are identified: market growth and learning rate variability.
- A revenue sharing contract coordinates the two-period supply chain, i.e. it restores efficiency.
- People involved in this research
  - Colleagues: Alain Bensoussan, Metin Cakanyildirim, Alp Muharremoglu Anyan Qi, Shouqiang Wang
  - Students: Shaokuan Chen, Tao Li, Anshuman Chutani, Ting Luo, Xi Shan
  - Staff Assistant: Lindsay Wilson

My Published Work on the Topic

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